1. Find the volume of the swimming pool shown in the diagram below:
   * Answer must include units.

Answer: \( 3240 \text{ ft.}^3 \)

Solution:

26 – (8 + 10) = 8 ft.
Either Pythagorean Theorem may be used to find the third side to the triangle (6 ft.), or Pythagorean Triple 6-8-10.

\[
\text{Area}_A = l \cdot w = 3(26) = 78 \text{ ft}^2
\]

\[
\text{Area}_B = \frac{1}{2} h(b_1 + b_2) \\
= \frac{1}{2} 6(10 + 18) \\
= 3(28) \\
= 84 \text{ ft}^2
\]

Total Area of Side(base) = \( \text{Area}_A + \text{Area}_B \)
= 78 ft\(^2\) + 84 ft\(^2\)
= 162 ft\(^2\)

Volume = area of side(base) \cdot height
= 162 ft\(^2\) \cdot 20 ft.
= 3240 ft\(^3\)
2. If the decimal 0.726874726874 repeats in this pattern indefinitely, what will be the result if the 82\textsuperscript{nd} digit is subtracted from the 83\textsuperscript{rd} digit?

Answer: -1

Solution: Find how many times the decimal repeats to get to the 82\textsuperscript{nd} team by dividing 82 by 6 = 13 with a remainder of 4. This means that the 82\textsuperscript{nd} digit will be 8 and the 83\textsuperscript{rd} digit will be 7. 7 – 8 = -1.
3. Stephen drove to his aunt’s house at 60mph. He made the return trip, over the same roadway, at 40mph. What was Stephen’s average speed for the whole trip? * Answer must include units.

Answer: 48mph

Solution: Find a consistent distance since the trip is the same distance both ways. In this case a distance of 120 miles makes sense (multiple of both original speeds).

\[
\frac{60m}{1hr} = \frac{120m}{2hrs} \quad \text{and} \quad \frac{40m}{1hr} = \frac{120m}{3hrs}
\]

Therefore, for a 120 mile trip, it would take 2 hours to get there (at 60 mph), and 3 hours to get back (at 40 mph).

The average speed for the total trip would be \( \frac{240m}{5hrs} = \frac{48m}{1hr} \); 48 mph
A family went on a vacation. One fifth of the family went to the beach, one third swam in the pool, and three times as many as the difference of these numbers stayed inside and worked on a puzzle. One family member just stayed in his room and read a book. How many people are in this family?

Answer: 15 people in this family

Solution:
\[
\frac{1}{5}x + \frac{1}{3}x + 3\left(\frac{1}{3} - \frac{1}{5}\right)x + 1 = x
\]
\[
\frac{1}{5}x + \frac{1}{3}x + 3\left(\frac{2}{15}\right)x + 1 = x
\]
\[
\frac{3}{15}x + \frac{5}{15}x + \frac{6}{15}x + 1 = x
\]
\[
\frac{14}{15}x + 1 = x
\]
\[
1 = \frac{1}{15}x
\]
\[
15 = x
\]
5. Square $ABCD$ has vertex $A$ at the center of a circle $A$. $AE = 10$ inches. What is the area of triangle $BCD$?  
* Answer must include units.

Answer: 25 in$^2$

Solution:

$AE = AC = 10$ inches (radius)

This means that $AC$, the diagonal of the square $ABCD = 10$ inches. Because this forms a special triangle $(90^\circ, 45^\circ, 45^\circ)$, or by Pythagorean theorem $(x^2 + x^2 = 10^2)$

Side length of square $ABCD = 5\sqrt{2}$.

Area of the square = $s^2 = (5\sqrt{2})^2 = 50$ in$^2$

Area of triangle $BCD$ is half the area of square $ABCD$, so:

Area of triangle $BCD = 25$ in$^2$
6. A hat has 20 cards in it. On each card is written a name of one of three people. Mike has twice as many cards with his name than Jane, and Jane as 4 more cards than Eric. If one card is randomly selected and not replaced, then a second card is randomly selected, what is the probability that the cards will have the names Mike and Eric on them?

* Give answer as a reduced fraction.

Answer: \( \frac{12}{95} \)

Solution:
Of the 20 cards in the hat, the number of cards for Mike (M), Jane (J), and Eric (E), are as follows:

\[
\begin{align*}
E & = 4 + E \\
J & = 4 + E \\
M & = 2J = 2(4+E)
\end{align*}
\]

The total number of cards in the hat is 20, so:

\[
E + J + M = 20
\]

Therefore,

\[
E + 4 + E + 2(4+E) = 20
\]

\[
E + 4 + E + 8 + 2E = 20
\]

\[
4E + 12 = 20
\]

\[
\begin{array}{c}
4E + 12 = 20 \\
\hline
-20 & -12 \\
\hline
4E & = 8 \\
\hline
E & = 2 \\
J & = 4 + E = 4 + 2 = 6 \\
M & = 2J = 2(6) = 12
\end{array}
\]

The probability that the cards will have the names Mike and Eric, will be the following:

\[
P[(M \text{ and } E) \text{ or } (E \text{ and } M)]
\]

\[
= \frac{12}{20} \cdot \frac{2}{19} + \frac{2}{20} \cdot \frac{12}{19}
\]

\[
= \frac{3}{5} \cdot \frac{2}{19} + \frac{3}{5} \cdot \frac{2}{19}
\]

\[
= \frac{6}{95} + \frac{6}{95}
\]

\[
= \frac{12}{95}
\]
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7. Frank, Jim, and Al all went to the store to buy soda, King Size Reese’s Cups, and magazines. Frank bought one bottle of soda, one package of Reese’s Cups, and two magazines, for $18. Jim bought two bottles of soda, three packages of Reese’s Cups, and one magazine, for $18. Al purchased five bottles of soda, two packages of Reese’s Cups, and four magazines, for $39. What was the price of each item purchased?
* Give answer as a reduced fraction.
Answer: Pop = $1, Reeses = $3, and Magazine = $7

Solution:

\[ \begin{align*}
1P + 1R + 2M &= 18 \\
2P + 3R + 1M &= 18 \\
5P + 2R + 4M &= 39 \\
2(1P + 1R + 2M &= 18) \\
-(2P + 3R + 1M &= 18) \\
\hline
-1R + 3M &= 18
\end{align*} \]

\[ \begin{align*}
5(2P + 3R + M &= 18) \\
-2(5P + 2R + 4M &= 39) \\
\hline
11R - 3M &= 12
\end{align*} \]

\[ \begin{align*}
-R + 3M &= 18 \\
11R - 3M &= 12 \\
\hline
10R &= 30 \\
R &= 3
\end{align*} \]

\[ \begin{align*}
-R + 3M &= 18 \\
-3 + 3M &= 18 \\
3M &= 21 \\
M &= 7
\end{align*} \]

\[ \begin{align*}
2P + 3R + M &= 18 \\
2P + 3(3) + 7 &= 18 \\
2P + 9 + 7 &= 18 \\
2P + 16 &= 18 \\
2P &= 2 \\
P &= 1
\end{align*} \]
8. In right triangle $ABC$, with right angle $ACB$, $AC = 8$ inches and $BC = 6$ inches. Point $X$ is on $AC$, equidistant from $A$ and $B$. Find $CX$.

* Give answer as a reduced fraction, or decimal.
* Answer must include units.

Answer: $\frac{7}{4}$ inches; $1\frac{3}{4}$ inches; 1.75 inches

Solution: Let $CX = y$. Then

$BX = AX = 8 - y$,

so by the Pythagorean theorem,

$t^2 + 36 = (8 - t)^2$

$= 64 - 16t + t^2$,

which implies that

$16t = 28$

so

$t = \frac{7}{4}$ inches; $1\frac{3}{4}$ inches; 1.75 inches
9. Three squares have the dimensions indicated in the diagram below. What is the area of the shaded quadrilateral?
   * Give answer as a reduced fraction, or decimal.
   * Answer must include units.

Answer: \( \frac{21}{4} \text{ m}^2; \ 5 \frac{1}{4} \text{ m}^2; \ 5.25 \text{ m}^2 \)

Solution:
Using similar triangles:

The lengths \( EB \) and \( FC \) need to be determined in order to calculate the area of \( \triangle ABE \) and \( \triangle ACF \).
\( \triangle ACF \) and \( \triangle ADG \) are similar, therefore:

\[
\frac{AC}{AD} = \frac{CF}{DG}
\]

\[
\frac{5}{10} = \frac{1}{2}
\]

Therefore,

\[
CF = \frac{1}{2}DG
\]

\[
= \frac{1}{2}(5)
\]

\[
= \frac{5}{2}
\]

Similarly, \( \triangle ABE \) and \( \triangle ACF \) are similar, therefore:

\[
\frac{AB}{AC} = \frac{BE}{CF}
\]
Therefore,

\[ BE = \frac{2}{5} CF \]

\[ = \frac{2 \left( \frac{5}{2} \right)}{\frac{5 \cdot 5}{2}} \]

\[ = 1 \]

Area of shaded quadrilateral would be equal to the following:

Area of \( \triangle ACF - \triangle ABE \)

\[ = \frac{1}{2} \left( \frac{5 \cdot 5}{2} \right) - \frac{1}{2} \left( 2 \cdot 1 \right) \]

\[ = \frac{25}{4} - 1 \]

\[ = \frac{21}{4} \text{ m}^2; \quad 5 \frac{1}{4} \text{ m}^2; \quad 5.25 \text{ m}^2 \]
10. Assume there is a rectangular barn, 8 ft. by 20 ft., which is surrounded by grass. If a goat is on a 20 ft. leash that is attached to a stake at the center of the 20 ft. side of the barn, how many square feet of grazing area will it have?

* Express in terms of \( \pi \).

**Answer:** \( 252\pi \) ft.\(^2\)

**Solution:**

Since area A is \( \frac{1}{2} \) of a circle, then

\[
\text{Area}_A = \frac{1}{2} \pi r^2
\]

\[
= \frac{1}{2} \pi \times 20^2
\]

\[
= \frac{1}{2} \pi \times 400
\]

\[
= 200\pi \text{ ft.}^2
\]
Since each area B is $\frac{1}{4}$ of a circle, then the area of both together is $\frac{1}{2}$ of a circle:

$$\text{Area}_B = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \pi 10^2$$

$$= \frac{1}{2} \pi 100$$

$$= 50\pi \text{ ft.}^2$$

Since each area C is $\frac{1}{4}$ of a circle, then the area of both together is $\frac{1}{2}$ of a circle:

$$\text{Area}_C = \frac{1}{2} \pi r^2$$

$$= \frac{1}{2} \pi 2^2$$

$$= \frac{1}{2} \pi 4$$

$$= 2\pi \text{ ft.}^2$$

Total Area = Area$_A$ + Area$_B$ + Area$_C$

$$= 200\pi + 50\pi + 2\pi$$

$$= 252\pi \text{ ft.}^2$$