AP Calculus AB

Syllabus

Course Overview and Philosophy

We cover all topics in the Calculus AB topic outline in the AP Calculus Course Description, and we also include integration by parts, shell method, and L'Hopital's rule for indeterminant forms. Our goal is not only for the students to perform well on the AP test as well as any future courses in mathematics, but also to instill our students with a sense of curiosity whereby they search for the meaning and prove the theorems underlying the mathematics and a sense of confidence to explain it to each other, a will to discover, explore, and learn new material, and a sense of appreciation and understanding of the aesthetics involved in grasping Calculus concepts.

Course Planner:

Chapter P – Preparation for Calculus (6 days)

- P.1 Graphs and Models
  Sketch the graph of an equation.
  Find the intercepts of a graph.
  Test a graph for symmetry with respect to an axis and the origin.
  Find the points of intersection of two graphs.
  Interpret mathematical models for real-life data.
- P.2 Linear Models and Rates of Change
  Find the slope of a line passing through two points.
  Write the equation of a line with a given point and slope.
  Interpret slope as a ratio or as a rate in a real-life application.
  Sketch the graph of a linear equation in slope-intercept form.
  Write equations of lines that are parallel or perpendicular to a given line.
- P.3 Functions and Their Graphs
  Use function notation to represent and evaluate a function.
  Find the domain and range of a function.
  Sketch the graph of a function.
  Identify different types of transformations of functions.
  Classify functions and recognize combinations of functions.

In this opening preparatory chapter, we allow the students opportunities to work with functions in different ways – graphically, numerically, analytically, and verbally, and we stress the advantages and disadvantages of each way. This way, students can see how if they need a rough idea of when one particular function is greater than another, they know that observing the graph might be best, but when finding exact points of intersection, an algebraic analysis is more powerful. In this chapter students are introduced to the idea that if they can explain the material to each other verbally, then they truly understand it.

Chapter 1 – Limits and Their Properties (10 days)

- 1.1 A Preview of Calculus
  Understand what calculus is and how it compares with precalculus.
  Understand that the tangent line problem is basic to calculus.
  Understand that the area problem is also basic to calculus.
- 1.2 Finding Limits Graphically and Numerically
  Estimate a limit using a numerical or graphical approach.
  Learn different ways that a limit can fail to exist.
  Study and use a formal definition of limit.
- 1.3 Evaluating Limits Analytically
  Evaluate a limit using properties of limits.
  Develop and use a strategy for finding limits.
  Evaluate a limit using dividing out and rationalizing techniques.
  Evaluate a limit using the Squeeze Theorem.
- 1.4 Continuity and One-Sided Limits
  Determine continuity at a point and continuity on an open interval.
  Determine one-sided limits and continuity on a closed interval.
  Use properties of continuity. Understand and use the Intermediate Value Theorem.
- 1.5 Infinite Limits
  Determine infinite limits from the left and from the right.
  Find and sketch the vertical asymptotes of the graph of a function.
- 3.5 Limits at Infinity
  Determine (finite) limits at infinity.
  Determine the horizontal asymptotes, if any, of the graph of a function.
  Determine infinite limits at infinity.

In this chapter, we not only graphically and numerically observe limits, and algebraically solve limit problems, but we also give students the opportunity to verbally discuss the ideas of the infinite and the infinitesimal. Because of this, we are able to emphasize the connections among these representations.

Chapter 2 – Differentiation (17 days)

- 2.1 The Derivative and the Tangent Line Problem
  Find the slope of the tangent line to a curve at a point.
  Use the limit definition to find the derivative of a function.
  Understand the relationship between differentiability and continuity.
- 2.2 Basic Differentiation Rules and Rates of Change
  Find the derivative of a function using the Constant Rule.
  Find the derivative of a function using the Power Rule.
  Find the derivative of a function using the Constant Multiple Rule.
  Find the derivative of a function using the Sum and Difference Rules.
  Find the derivative of the sine function and of the cosine function.
  Use derivatives to find rates of change.
- 2.3 The Product and Quotient Rules and Higher-Order Derivatives
  Find the derivative of a function using the Product Rule.
  Find the derivative of a function using the Quotient Rule.
  Find the derivative of a trigonometric function.
  Find a higher-order derivative of a function.
- 2.4 The Chain Rule
  Find the derivative of a composite function using the Chain Rule.
  Find the derivative of a function using the General Power Rule.
  Simplify the derivative of a function using algebra.
  Find the derivative of a trigonometric function using the Chain Rule.
- 2.5 Implicit Differentiation
  Distinguish between functions written in implicit form and explicit form.
  Use implicit differentiation to find the derivative of a function.
- 2.6 Related Rates
  Find a related rate.
  Use related rates to solve real-life problems.

In chapter 2, we introduce the idea of instantaneous rate of change and define the derivative to mean two things. The first understanding of the derivative is as the instantaneous rate of change of a relation at a point, and the second understanding of the derivative is of a new relation or function that describes another relation's or function's rate of change. We begin by sketching the graph of a position function of a jogger on the board, for which we ask students to estimate the jogger's speed numerically at various points in time. We then have the students come to the board and plot points on a velocity graph to represent the derivative graphically. At first, all we can do is use the regression capabilities of the graphing calculator to determine the algebraic equation of the derivative. Occasionally, we return to the problem to create conjectures about the derivatives of power and trigonometric functions. We have the students write their own related rates problems, and then have them give them to each other in pairs to see if they've understood not only how to do these type of problems but also what kind of information is necessary to having them be solvable. The students are extremely creative when we do this, some writing their problems involving serious issues, such as tsunami wave speed and radiation fallout, and some writing their problems in fun ways, like how quickly Napoleon Dynamite needs to let out his dental floss to have the action figure trailing the bus go 22 mph, and how much blueberry juice comes out of Violet Beauregard if the Oompaloompas squeeze 20 sq. feet of surface area each second.

Chapter 3 – Applications of Differentiation (17 days)

- 3.1 Extrema on an Interval
  Understand the definition of extrema of a function on an interval.
  Understand the definition of relative extrema of a function on an open interval.
  Find extrema on a closed interval.
- 3.2 Rolle's Theorem and the Mean Value Theorem
  Understand and use Rolle's Theorem.
  Understand and use the Mean Value Theorem.
- 3.3 Increasing and Decreasing Functions and the First Derivative Test
  Determine intervals on which a function is increasing or decreasing.
  Apply the First Derivative Test to find relative extrema of a function.
- 3.4 Concavity and the Second Derivative Test
  Determine intervals on which a function is concave upward or concave downward.
  Find any points of inflection of the graph of a function.
  Apply the Second Derivative Test to find relative extrema of a function.
- 3.6 A Summary of Curve Sketching
  Analyze and sketch the graph of a function.
- 3.7 Optimization Problems
  Solve applied minimum and maximum problems.
- 3.8 Newton's Method
  Approximate a zero of a function using Newton's Method.
- 3.9 Differentials
  Understand the concept of a tangent line approximation.
  Compare the value of differential, dy, with the actual change in y,

Estimate a propagated error using a differential.
Find the differential of a function using differentiation formulas.

Throughout Chapter 3 we emphasize the meaning of the derivative as a rate of change; we ask the students to determine what the units of measurement in the derivative will be before actually doing any problems. Students are asked to design, decorate, and create a Calculus based cereal box from a piece of graph paper and explain why it holds a maximum volume.

Chapter 4 – Integration (20 days)

- 4.1 Antiderivatives and Indefinite Integration
  Write the general solution of a differential equation.
  Use indefinite integral notation for antiderivatives.
  Use basic integration rules to find antiderivatives.
  Find a particular solution of a differential equation.
- 4.2 Area
  Use sigma notation to write and evaluate a sum.
  Understand the concept of area.
  Approximate the area of a plane region.
  Find the area of a plane region using limits.
- 4.3 Riemann Sums and Definite Integrals
  Understand the definition of a Riemann sum.
  Evaluate a definite integral using limits.
  Evaluate a definite integral using properties of definite integrals.
- 4.4 The Fundamental Theorem of Calculus
  Evaluate a definite integral using the Fundamental Theorem of Calculus.
  Understand and use the Mean Value Theorem for Integrals.
  Find the average value of a function over a closed interval.
  Understand and use the Second Fundamental Theorem of Calculus.
- 4.5 Integration by Substitution
  Use pattern recognition to find an indefinite integral.
  Use a change of variables to find an indefinite integral.
  Use the General Power Rule for Integration to find an indefinite integral.
  Use a change of variables to evaluate a definite integral.
  Evaluate a definite integral involving an even or odd function.
- 4.6 Numerical Integration
  Approximate a definite integral using the Trapezoidal Rule.

To help our students fully understand the integral, we show them examples of how area under a velocity curve can capture an amount of distance traveled using a jaguar as an example, since a jaguar's velocity can vary greatly through time. We show the students how to determine the idea of the definite integral graphically through area, numerically through approximation techniques on a table of values, and algebraically through the Fundamental Theorem of Calculus. We spend 2 days returning to problems that we've done using sigma notation or general approximation and apply the Fundamental Theorem of Calculus all over to help strengthen their ability and comfort with it. We have students explain and write essays explaining its use in determining definite integrals.

Chapter 5 – Logarithmic, Exponential, and Other Transcendental Functions (23 days)

- 5.1 The Natural Logarithmic Function: Differentiation
  Develop and use properties of the natural logarithmic function.
  Understand the definition of the number e.
  Find derivatives of functions involving the natural logarithmic function.
- 5.2 The Natural Logarithmic Function: Integration
  Use the Log Rule for Integration to integrate a rational function.
  Integrate trigonometric functions.

Project Focus: The Most Powerful Force in the Universe:
After the first two sections of Chapter 5, we give the students a project where they are to examine the ideas of compound interest (claimed by Einstein to be the most powerful force in the universe). We ask them to graph several exponential curves given differing interest rates, along with their derivatives. By a discovery approach, they are to make a conjecture about exponential curves and their derivatives (that they are similar curves, that is, that the derivative of an exponential function is itself an exponential function). We then ask them to explore (using their graphing calculators or WinPlot or Mathematica) different interest rates and see if they can find an exponential function whose derivative is equal to itself. We allow the students an opportunity to discuss the results of their projects with each other, and have them present their findings to each other. We've had students use the idea of logarithmic differentiation to prove their findings.

- 5.3 Inverse Functions
  Verify that one function is the inverse function of another function.
  Determine whether a function has an inverse function.
  Find the derivative of an inverse function.
- 5.4 Exponential Functions: Differentiation and Integration
  Develop properties of the natural exponential function.

Differentiate natural exponential functions.
Integrate natural exponential functions.

- 5.5 Bases Other than e and Applications
  Define exponential functions that have bases other than e.
  Differentiate and integrate exponential functions that have bases other than e.
  Use exponential functions to model compound interest and exponential growth.
- 5.6 Inverse Trigonometric Functions: Differentiation
  Develop properties of the six inverse trigonometric functions.
  Differentiate an inverse trigonometric function.
  Review the basic differentiation formulas for elementary functions.
- 5.7 Inverse Trigonometric Functions: Integration
  Integrate functions whose antiderivatives involve inverse trigonometric functions.
  Use completing the square to integrate a function.
  Review the basic integration formulas involving elementary functions.

Chapter 6 – Differential Equations (8 days)

- 6.1 Slope Fields and Euler's Method
  Use initial conditions to find particular solutions of differential equations.
  Use slope fields to approximate solutions of differential equations.
  Recognize and solve differential equations that can be solved by separation of variables.
- 6.2 Differential Equations: Growth and Decay
  Use separation of variables to solve a simple differential equation.
  Use exponential functions to model growth and decay in applied problems.

For these sections, we give the students a differential equation and project a blank WinPlot grid onto the board. We give each student a small whiteboard with a couple of coordinate pairs and ask them to compute a slope using the differential equation. Each student has an opportunity to come to the board and draw a small segment representing their slope using the given coordinate point as the midpoint of their segment, thus creating a slope field. Then we use WinPlot's built-in slope field grapher to check the results. We also ask individual students to draw particular solutions through given points by following the curvature of the slopes of the slope field. We also help the students to write a program for their graphing calculator that creates slope fields from differential equations.

Chapter 7 – Applications of Integration (20 days)

- 7.1 Area of a Region Between Two Curves
  Find the area of a region between two curves using integration.

Find the area of a region between intersecting curves using integration.
Describe integration as an accumulation process.

- 7.2 Volume: The Disk Method
  Find the volume of a solid of revolution using the disk method.
  Find the volume of a solid of revolution using the washer method.
  Find the volume of a solid with known cross sections.
- 7.3 Volume: The Shell Method
  Find the volume of a solid of revolution using the shell method.
  Compare the uses of the disk method and the shell method.
- 7.4 Arc Length and Surfaces of Revolution
  Find the arc length of a smooth curve.
  Find the area of a surface of revolution.
- 7.5 Work
  Find the work done by a constant force.
  Find the work done by a variable force.

We use WinPlot and Mathematica notebooks to give the students a representation of solids of revolution and help them to determine appropriate cross-sectional shapes to find volumes. We ask the students to come up with ideas for what the shapes might represent (dog bowls, trumpets, stadiums, etc.) to help make the learning more meaningful to them.

Student Evaluation

Every chapter, we assess the students cumulatively in small increments. For example, in chapter 7, we give a mini-quiz on areas bounded between curves, then we give a quiz on areas and volumes of solids of revolution, then a quiz on area, volume, arc length and surface area, and finally a chapter test. This allows students a chance to not only demonstrate knowledge of the new material but also to show how well they've retained previous knowledge, reinforces the ideas of the chapter by showing connections between problems, and strengthens their understanding and ability to transfer the concepts in new and interesting ways.

Quizzes and tests have some problems similar to given examples and some problems not similar to given examples. This way we can assess how well students have learned the material, but another side effect is that students gain an appreciation for the diversity in which the concepts can be applied. We vary the types of problems, including multiple-choice problems and matching problems, and also short answer and essay questions, for which we ask students to write their answers in complete sentences.

Towards the end of each semester, we work on AP Released Exams. A set of appropriate problems is chosen each day as homework, and students may work together and also come to us for guidance. In April, we hold practice mini-exams during class which we collect, grade, return to the students, and then go over by asking the students to present their work on either the overhead or the chalkboard. Sometimes, students ask to demonstrate their ideas using the graphing calculator or WinPlot. We alternate between free-response problems and multiple-choice problems until the AP Exam is administered.

Primary Textbook

Larson, Ron, Robert P. Hostetler, and Bruce H. Edwards. *Calculus of a Single Variable*, - 8th ed., Boston, New York: Houghton Mifflin

Technology and Computer Software

We discuss the nuances of the built-in functions of the calculator with our students; we make sure they know that frequently when the calculator is finding a root (or a point of intersection, or even a numerical derivative), it is an iterative process that sometimes leads to a slight numerical error, whereas an analytical approach will find exact solutions. We teach the students how to use the built-in functions of their calculators (such as Zero, Intersection, Regression, NDeriv, and FnInt). We use a TI-Presenter in the classroom as well as a TI-83 poster to show certain keystrokes.

We use computer software programs WinPlot and Mathematica to help our students visualize solids of revolution, and we've created electronic notebooks that demonstrate visually the processes of revolution and slicing into representative pieces that the integral can accumulate. We ask students to play with these programs and to have fun in both formal assignments and projects involving discovery approaches. We've also had students use their school internet access to research different ideas when we give them open-ended writing assignments (for example, a student found a function online describing a tsunami waves' speed as a function of the depth of water, and wrote a detailed related rates question relating the speed of the tsunami to the rate that the depth of water is changing).